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A Current Algebra Calculation of the π - π Scattering Lengths.

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Summary. — A new approach is presented for the calculation of the π - π scattering lengths based on current algebra. The parametrization chosen for the amplitude allows for the elastic unitarity cuts. A family of solutions has been found which contains Weinberg's results.

The current algebra techniques have been extensively used during the last months for the calculation of the pion-pion s -wave scattering lengths ⁽¹⁻⁶⁾. However, in all cases, one had to make additional assumptions and the purpose of this paper is to try to investigate how sensitive the results were on these extra assumptions.

We shall follow mainly the method introduced by WEINBERG ⁽³⁾ and we shall try to weaken some of his assumptions.

Let us consider the expression

$$(1) \quad (2\pi)^4 \delta^4(\sum k_i) F_\pi^4 m_\pi^2 M =$$

$$= -i \int d^4x \langle 0 | T(\partial_\mu A_\mu^a(x_1), \partial_\nu A_\nu^e(x_2), \partial_\rho A_\rho^b(x_3), \partial_\sigma A_\sigma^d(x_4)) | 0 \rangle \cdot$$

$$\cdot \prod_{i=1}^4 ((k_i^2 + m_\pi^2) \exp[-ik_i x_i]) ,$$

⁽¹⁾ A. P. BALANCHANDRAN, M. G. GUNDZIK and F. NICODEMI: to be published.

⁽²⁾ Y. TOMOZAWA: to be published.

⁽³⁾ S. WEINBERG: *Phys. Rev. Lett.*, **17**, 616 (1966).

⁽⁴⁾ N. N. KHURI: *Phys. Rev.*, **153**, 1477 (1967).

⁽⁵⁾ T. AKIBA and K. KANG: to be published.

⁽⁶⁾ J. SUCHER and C.-H. WOO: to be published.

where d^4x stands for $d^4x_1 \dots d^4x_4$ and a, b, c and d are the isospin indices. F_π is the pion decay coupling constant defined by

$$(2) \quad \langle 0 | \partial_\mu A_\mu^a | \pi_k^b \rangle = \frac{F_\pi m_\pi^2 \delta_{ab}}{\sqrt{(2\pi)^3 2k_0}}.$$

M , as defined by (1), is a function of six variables and it is symmetric under the exchange of any pair of isospin indices together with the corresponding k 's. Furthermore, at $k_i^2 = -m_\pi^2$ ($i = 1, \dots, 4$), M is proportional to the physical π - π scattering amplitude. It is convenient to write M in the form usually used for the π - π amplitude, *i.e.*

$$(3) \quad M = \delta_{ab} \delta_{cd} M_1 + \delta_{ab} \delta_{cd} M_2 + \delta_{ab} \delta_{cd} M_3,$$

where M_1 , M_2 and M_3 are functions which have the correct symmetry properties under the exchange of any pair of axial spurions.

The important thing about M is that its value is known in some particular points:

a) At $k_1^2 = k_2^2 = k_3^2 = -m_\pi^2$ M is given by

$$(4) \quad (2\pi)^4 \delta^4(\sum k_i) F_\pi^2 m_\pi^2 M = \\ = -i \int d^4x_4 \langle \pi_1^a(-k_1), \pi_2^c(-k_2); \text{out} | \partial_\sigma A_\sigma^d(x_4) | \pi_3^b(k_3) \rangle (k_4^2 + m_\pi^2) \exp[-ik_4 x_4] = \\ = k_4^\sigma \int d^4x_4 \langle \pi_1^a, \pi_2^c; \text{out} | A_\sigma^d(x_4) | \pi_3^b \rangle (k_4^2 + m_\pi^2) \exp[-ik_4 x_4],$$

where, in the last step, the translational properties of the matrix element have been used. Equation (4) shows clearly that M vanishes when $k_4 = 0$. This is the well-known Adler's self-consistency condition⁽⁷⁾. In terms of M_1 , M_2 and M_3 it reads

$$(5) \quad M_1 = M_2 = M_3 = 0, \quad \text{for} \quad k_1^2 = k_2^2 = k_3^2 = -m_\pi^2, \quad k_4 = 0.$$

b) At $k_2^2 = k_4^2 = -m_\pi^2$, M can be written as

$$(6) \quad (2\pi)^4 \delta^4(\sum k_i) F_\pi^2 m_\pi^4 M = \\ = -i \int d^4x_1 d^4x_3 \langle \pi_2^c(-k_2) | T(\partial_\mu A_\mu^a(x_1), \partial_\nu A_\nu^b(x_3)) | \pi_4^d(k_4) \rangle \cdot \\ \cdot (k_1^2 + m_\pi^2)(k_3^2 + m_\pi^2) \exp[-ik_1 x_1] \exp[-ik_3 x_3].$$

(7) S. L. ADLER: *Phys. Rev.*, **137**, B 1022 (1965); **139**, B 1638 (1965).

By applying on eq. (6) the standard current algebra manipulations, one can show ⁽³⁾ that, for $k_1 = -k_3 \rightarrow 0$

$$(7) \quad \frac{1}{2} (M_2 - M_3) = -8 \left(\frac{1}{F_\pi} \right)^2 (k_1 \cdot k_2)$$

and

$$(8) \quad M_2 + M_3 = 0.$$

In deriving eq. (7) use has been made of the commutation relation

$$(9) \quad \delta(x_0 - y_0) [A_0^a(x), A_\mu^b(y)] = 2i\varepsilon_{abc} V_\mu^c(x) \delta^4(x - y).$$

Equation (8) follows from the commutation relation

$$(10) \quad \delta(x_0 - y_0) [A_0^a(x), \partial_\mu A_\mu^b(y)] = \delta_{ab} \sigma(x) \delta^4(x - y),$$

where $\sigma(x)$ is an unspecified local, scalar operator. In deriving eq. (8), only the isospin structure of the commutation relation (10) is required. However, we want to emphasize that eq. (10) represents a further assumption, outside the usual framework of current algebra, although it is true in some particular models. Physically eq. (10) means that, at the point $k_2^2 = k_4^2 = -m_\pi^2$ and $k_1 = k_3 = 0$, the $I = 2$ part of the amplitude in the t -channel vanishes. It would be interesting to investigate the influence of eq. (8) on the values of the scattering lengths. In this case (10) should be replaced by a more general assumption, namely that the commutator is symmetric in a and b , but not necessarily proportional to δ_{ab} , allowing for an $I = 2$ nonzero part of the amplitude in the t -channel. Unfortunately, we have not been able to find any other relation to replace eq. (8) and, with our parametrization of the amplitude, we cannot afford to leave out any of the equations.

The system of equations (5), (7) and (8) gives the maximum information that current algebra can provide us. As we see, none of this information concerns the physical points. One could, of course, assume that, as in the π - N case, the amplitude does not vary significantly when one or two of the pions go to zero energy momentum, but for the π - π amplitude this is a rather dubious assumption and furthermore, as we shall see later, it may lead to inconsistencies. Therefore, a reasonable way to continue is to assume a parametrization of M and to use these equations to fix the arbitrary parameters. WEINBERG ⁽³⁾ has assumed that M can be expanded in powers of s , t and u , and one can neglect all powers higher than the first ⁽⁸⁾. Therefore, he assumes the fol-

⁽⁸⁾ N. N. KHURI in ref. ⁽⁴⁾ has considered also the second-order terms by adding some further assumptions.

lowing form for M :

$$(11) \quad M = \delta_{ab}\delta_{cd}[A + B(s + u) + Ct + \dots] + \text{permutations},$$

where $s = -(k_1 + k_2)^2$, $t = -(k_1 + k_3)^2$, $u = -(k_1 + k_4)^2$ and A , B and C are constants to be determined by the three equations above. In Weinberg's original approach (11) was assumed to be valid up to and somewhat beyond the physical threshold. However, such an assumption clearly violates the unitarity condition. Therefore, Weinberg has remarked that his small scattering lengths could be due to the fact that, in writing (11), one had already assumed them to be negligible so that, strictly speaking, current algebra does not prove them to be small. Thus, it is interesting to investigate what happens if one does not assume that M , as a function of s , t and u , is analytic at threshold. There are several ways to avoid this assumption. In this paper we shall do it by replacing eq. (11) by a more flexible parametrization which allows for the elastic unitarity cuts ⁽⁹⁾. As it is well known, these cuts are of the square-root type, so, let us introduce the variables

$$(12) \quad k_s = \frac{1}{2}\sqrt{4-s}, \quad k_t = \frac{1}{2}\sqrt{4-t}, \quad k_u = \frac{1}{2}\sqrt{4-u}.$$

In terms of these variables M has a much larger analytic domain which contains the physical threshold. Note that the symmetry properties of M do not allow for any terms proportional to k_i^2 other than the ones already contained in s , t and u . On the other hand, if one neglects the singularities of M in the k_i^2 variables, it follows that, up to second order in k_s , k_t and k_u , M is a function of only those three variables. Therefore M is assumed to have the following form:

$$(13) \quad M = \delta_{ab}\delta_{cd}[A + B(k_s + k_u) + Ck_t + Dk_t^2 + E(k_s^2 + k_u^2) + Fk_sk_u + Gk_t(k_s + k_u)] + \\ + \delta_{ad}\delta_{cb}[A + B(k_s + k_t) + Ck_u + Dk_u^2 + E(k_s^2 + k_t^2) + Fk_sk_t + Gk_u(k_s + k_t)] + \\ + \delta_{ac}\delta_{bd}[A + B(k_t + k_u) + Ck_s + Dk_s^2 + E(k_t^2 + k_u^2) + Fk_tk_u + Gk_s(k_t + k_u)].$$

Equation (13) (or (11)) is a weak form of the so called PDDAC hypothesis ⁽¹⁰⁾ (pole dominance to the divergence of the axial current). In order to see it more clearly let us consider the matrix element of $\partial_\mu A_\mu$ between two

⁽⁹⁾ T. AKIBA and K. KANG in ref. (5) have followed a different method: eq. (11) has been assumed for the region below threshold and dispersion relations have been used to estimate the corrections due to the extrapolation up to the physical region. A third and very interesting alternative has been examined by A. DONNACHIE: to be published.

⁽¹⁰⁾ We thank Dr. J. BELL for very clarifying remarks concerning this point.

states $|a\rangle$ and $|b\rangle$

$$(14) \quad N = \langle a | \partial_\mu A_\mu | b \rangle.$$

Since N has the pion pole in the variable k^2 , where $k = p_a - p_b$, it can be written as

$$(15) \quad N(k^2) = \frac{R}{k^2 + m_\pi^2} + N_1(k^2),$$

where R is independent of k^2 and N_1 has no singularity at $k^2 = -m_\pi^2$. A rigorous application of PDDAC (hereafter called «restricted PDDAC») would mean that $N_1 = 0$. However, this assumption is too strong and furthermore it is meaningless unless one specifies which are the other variables. For example, suppose that $|a\rangle$ is a two-particle state and $|b\rangle$ a single-particle one. Then N is a function of three variables. It is clear that if the restricted PDDAC holds for one set of variables, say s , t and k^2 , it will certainly fail for some others, for example, s , u and k^2 , since $u = \sum m^2 + k^2 - s - t$, unless R is a constant with respect to all variables ⁽¹¹⁾. It is worth-while noticing that the formal replacement $\partial_\mu A_\mu = \mathcal{O}\varphi_\pi$ does not solve the problem at all, since the same ambiguity arises for the matrix elements of the operator φ_π for which eq. (15) holds with the same analytic properties. Therefore, a more meaningful approximation would be to assume that N_1 is a constant but not necessarily equal to zero ⁽¹²⁾. One can still give a meaning to the restricted PDDAC assumption ($N_1 = 0$) if one agrees to neglect all terms that are of the order of m_π^2 in the expression $(k^2 + m_\pi^2)N = R + (k^2 + m_\pi^2)N_1$. This can be the case, namely, if the states $|a\rangle$ and $|b\rangle$ contain heavy particles and the success of calculations, such as the π - N scattering lengths based on this restricted PDDAC hypothesis, seems to support this point of view. Note that restricted PDDAC implies that the matrix elements of the σ operator of eq. (10) vanish. These terms had, in fact, been neglected in the π - N case. In our case this would have given us a fourth equation expressing the vanishing of the σ term. The system of these four equations is incompatible with eq. (11). TOMOZAWA ⁽²⁾ has applied this method, together with the assumption of no significant variation of the amplitude with the external momenta, and ended with a result that was in conflict with Bose statistics. This inconsistency however, is not

⁽¹¹⁾ This has already been pointed out by several authors. See, for example, ref. (7), W. I. WEISBERGER: *Phys. Rev.*, **143**, 1302 (1966) and F. T. MEIERE and M. SUGAWARA: *Phys. Rev.*, **153**, 1709 (1967).

⁽¹²⁾ In terms of the residue function R , the restricted PDDAC means that R is a constant as we have already remarked. This new assumption would mean that, in a power series expansion of R with respect to the other variables, one could keep also linear terms.

due to his asymmetric way of reducing with respect to the two pions, but to a too strong PDDAC assumption. By using PDDAC, as given by eq. (15), with N_1 a constant, no inconsistency is found.

Let us come back to eq. (13). At the point $k_2^2 = k_4^2 = -m_\pi^2$, $k_1 = -k_3 \rightarrow 0$ we find

$$(16) \quad s = 1 - 2k_1 \cdot k_2, \quad t = 0, \quad u = 1 + 2k_1 \cdot k_2,$$

therefore

$$(17) \quad \begin{cases} k_s = \frac{\sqrt{3}}{2} - \frac{1}{4\sqrt{3}}(s-1) + \dots, \\ k_t = 1, \\ k_u = \frac{\sqrt{3}}{2} - \frac{1}{4\sqrt{3}}(u-1) + \dots, \end{cases}$$

where the units $h = c = m_\pi = 1$ have been used. In the same way we find that, at the point $k_1^2 = k_2^2 = k_3^2 = -m_\pi^2$, $k_4 = 0$

$$(18) \quad k_s = k_t = k_u = \frac{\sqrt{3}}{2}.$$

Using (13), (17) and (18), eqs. (5), (7) and (8) read

$$(19) \quad A + \sqrt{3}B + \frac{\sqrt{3}}{2}C + \frac{3}{4}D + \frac{3}{2}E + \frac{3}{4}F + \frac{3}{2}G = 0,$$

$$(20) \quad B - C + F - G + \sqrt{3}(E - D) = -19.1,$$

$$(21) \quad 2A + (\sqrt{3} + 2)B + \sqrt{3}C + \frac{3}{2}D + \frac{7}{2}E + \sqrt{3}F + \left(\sqrt{3} + \frac{3}{2}\right)G = 0.$$

In writing eq. (20) the value of F_π , given by (2), has been used ($1/F_\pi^2 = 0.69$).

In order to obtain some more information we use the unitarity of the partial waves

$$(22) \quad \frac{8\pi W}{q} \operatorname{Im} F_l^I(q) = |F_l^I(q)|^2,$$

where W equals $\sqrt{s}$ and q is the magnitude of the three-momentum in the c.m.s. F_l^I is the l partial wave for the I isospin state given in terms of phase shifts, by

$$(23) \quad F_l^I = \frac{8\pi W}{q} e^{i\delta_l^I} \sin \delta_l^I.$$

Of course, we cannot expect that a simple form like (13) could satisfy unitarity everywhere, so, we require (22) to be satisfied at threshold ⁽¹³⁾. In this way we obtain two equations, one for each isospin state of the s -wave

$$(24) \quad 2B + 3C + 2F + 8G = \frac{-1}{32\pi} (5A + 8B + 2C + 2D + 8E + 3F + 2G)^2.$$

$$(25) \quad B + F + G = \frac{-1}{16\pi} (A + B + C + D + E + G)^2.$$

Equations (24) and (25) come respectively from the $I=0$ and $I=2$ parts of the s -wave.

Finally, we use the known properties of the π - π amplitude obtained from axiomatic field theory. MARTIN ⁽¹⁴⁾ has proved the following relations for the $\pi^0\pi^0$ combination of the amplitude.

Let us define $F^{00}(s, t)$ to be the on-the-mass-shell $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ scattering amplitude given, in terms of the F^I 's by

$$(26) \quad F^{00}(s, t) \sim \left[\frac{1}{3} F^0(s, t) + \frac{2}{3} F^2(s, t) \right],$$

and let $F_0^{00}(s)$ be its s -wave projection. We shall use the following results proved by Martin:

$a)$ $F^{00}(s, t)$ has a minimum at the symmetric point $s = t = u = \frac{4}{3}$. The existence of an extremum at that point follows from crossing symmetry, so it is automatically satisfied if one assumes a parametrization like (11) or (13). Thus, the only information that we can get from this property is the sign of the second derivative

$$(27) \quad \left. \frac{\partial^2 F^{00}(s, t)}{\partial s^2} \right|_{s=t=\frac{4}{3}} > 0.$$

As we shall see all our solutions will satisfy (27). This was not *a priori* obvious since, to prove (27), one needs the positivity of the absorptive parts, not only at threshold but everywhere.

⁽¹³⁾ If we expand (22) in powers of q^2 only the zero-order term can be considered. In order to calculate the next term we need the q^3 terms of $\text{Im } F_I$. But these terms are unreliable since we have neglected third-order terms in the expansion (13).

⁽¹⁴⁾ A. MARTIN: *Nuovo Cimento*, **47**, 265 (1967). See also A. MARTIN and Y. S. JIN, *Phys. Rev.*, **135**, B 1369 (1964).

b) $F_0^{00}(s)$ and its derivatives satisfy the following inequalities:

$$(28a) \quad F_0^{00}(s) < F_0^{00}(4), \quad 0 \leq s < 4,$$

$$(28b) \quad F_0^{00}(0) \geq F_0^{00} \left(2 \left[1 + \frac{1}{\sqrt{3}} \right] \right),$$

$$(28c) \quad \frac{dF_0^{00}(s)}{ds} > 0, \quad 1.7 \leq s \leq 4,$$

$$(28d) \quad \frac{dF_0^{00}(s)}{ds} < 0, \quad 0 \leq s \leq 1.29.$$

In particular (28c) ⁽¹⁵⁾ and (28d) show that $F_0^{00}(s)/ds$ vanishes for s_0 between 1.29 and 1.7. In terms of our parameters $A, \dots, G$ this condition reads

$$(29) \quad \frac{d\Sigma_0}{ds} (2B + C) + \frac{d\Sigma\Sigma_0}{ds} (F + 2G) = 0, \quad s = s_0,$$

where $\Sigma = k_s + k_t + k_u$ and $\Sigma\Sigma = k_s k_t + k_t k_u + k_u k_s$ and Σ_0 and $\Sigma\Sigma_0$ are the s -wave projections of Σ and $\Sigma\Sigma$ respectively. After some simple algebra we find

$$(30) \quad \left. \frac{d\Sigma_0}{ds} \right|_{s=s_0} = -\frac{1}{4} \frac{1}{\sqrt{4-s_0}} + \frac{2}{3} \frac{\frac{1}{2}s_0^{\frac{3}{2}} - 6s_0^{\frac{1}{2}} + 8}{(s_0-4)^2},$$

$$(31) \quad \left. \frac{d\Sigma\Sigma_0}{ds} \right|_{s=s_0} = \frac{1}{3} \frac{s_0^{\frac{3}{2}} - 6s_0^{\frac{1}{2}} + 4}{(4-s_0)^{\frac{3}{2}}} - \frac{\sqrt{s_0}}{4(4-s_0)} + \frac{s_0^2 - 8s_0 - 48}{16(s_0-4)^2} \sin^{-1} \frac{s_0-4}{s_0+4},$$

s_0 varies between 1.29 and 1.7 but the results are practically independent (1% or less) of its precise value.

The scattering lengths a_0 and a_2 for the $I=0$ and $I=2$ s -waves respectively are given by the formulae

$$(32) \quad a_0 = -\frac{1}{32\pi} (5A + 8B + 2C + 2D + 8E + 3F + 2G),$$

$$(33) \quad a_2 = -\frac{1}{16\pi} (A + B + C + D + E + G).$$

The equations (19), (20), (21), (24), (25) and (29) form a system of six equations with seven unknowns, so we are not able to determine the scattering lengths exactly. The inequalities (27) and (28) have been found to be satisfied in the entire region of solutions. Therefore, we establish a relation between a_0 and a_2

⁽¹⁵⁾ The range of validity of (28c) has been extended in the region $1.76 \leq s \leq 2$, by A. D. COMMON (unpublished). We thank A. MARTIN for informing us about this work.

which is represented graphically by the hyperbola shown on the Fig. 1. The following remarks can be made from the inspection of this graph:

a) Weinberg's solution $a_0 = 0.2$ and $a_2 = -0.06$ belongs, with an excellent agreement, to our family of solutions (for $a_2 = -0.06$ we find $a_0 = 0.175$).

b) There is a forbidden range for a_2 , namely the gap between -0.46 and -1.9 . Since the left branch of the hyperbola can most probably be excluded on the basis of the available experimental results, we conclude that

$$(34) \quad a_2 > -0.46.$$

This limit can be further improved if we add the requirement, which is strongly supported by all experimental results, that a_0 should be positive

$$(34a) \quad a_2 \geq -0.14.$$

For a_0 no restrictions have been found.

c) If a_0 is large and positive of the order of 1.5, as some results for $\pi\text{-}\mathcal{N}$ scattering ⁽¹⁶⁾ or K_{e4} decay ⁽¹⁷⁾ seemed to indicate, a_2 must also be positive and quite appreciable of the order of 0.7 ⁽¹⁸⁾.

⁽¹⁶⁾ See, for example, J. HAMILTON: *Strong Interactions and High-Energy Physics*, Ed. R. G. MOORHOUSE (London, 1964). For a review of other experimental results see also N. E. BOOTH and A. ABASHIAN: *Phys. Rev.*, **132**, 2314 (1963) and especially Sect. 7. We must however remark that in all these analyses the formulae used did not allow for an $I=0$ s -wave low mass resonance (π particle). C. LOVELACE, R. M. HEINZ and A. DONNACHIE: *Phys. Lett.*, **22**, 332 (1966), have examined this possibility and they have found that, if one includes the σ resonance, the experimental results are consistent with small scattering lengths.

⁽¹⁷⁾ C. KACSER, P. SINGER and T. N. TRUONG: *Phys. Rev.*, **137**, B 1605 (1965); **139**, AB 5 (1965); N. CABIBBO and A. MAKSYMOWICZ: *Phys. Rev.*, **137**, B 438 (1965); J. ILIOPOULOS: *Nuovo Cimento*, **38**, 907 (1965); **39**, 413 (1965).

⁽¹⁸⁾ A. DONNACHIE, ref. ⁽⁹⁾, has arrived to similar conclusions. We wish to thank him for communicating his results to us prior to publication.

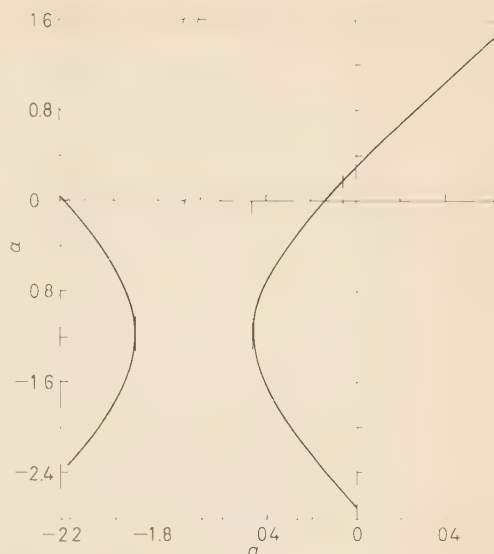


Fig. 1. - Plot of a_0 as a function of a_2 . The units are $\hbar = c = m_\pi = 1$.

d) For the whole physically interesting range of $a_0 > 0$ the right-hand side of the hyperbola can be approximated with a very good accuracy by the linear form

$$(35) \quad a_0 - 2a_2 = 0.28.$$

* * *

The author wishes to thank Dr. J. BELL for numerous suggestions and clarifying remarks, and Dr. J. D. BESSIS and Dr. A. MARTIN for very helpful discussions.

Note added in proofs.

Dr. A. MARTIN has pointed out to me that one can further restrict the arbitrariness of eq. (13) by imposing the correct threshold behaviour to the partial wave amplitudes combined with analyticity requirements. The result is that the parameters F and G in eq. (13) should vanish. These conditions allow us to discriminate between our family of solutions. There exist four points on the hyperbola of Fig. 1 for which $F = G = 0$. These are:

a) $a_0 \approx 0.175$, $a_2 \approx -0.06$ (Weinberg's solution);

b) $a_0 \approx -2.45$, $a_2 \approx -0.11$;

c) $a_0 \approx 0$, $a_2 \approx -2.3$;

d) $a_0 \approx -2.0$, $a_2 \approx -2.2$.

Further research is continued along these lines and the results will be reported elsewhere.

RIASSUNTO (*)

Si presenta un nuovo metodo, basato sull'algebra delle correnti, per calcolare le lunghezze dello scattering π - π . La parametrizzazione scelta per l'ampiezza tiene conto dei tagli di unitarietà elastici. Si è trovata una famiglia di soluzioni che contiene i risultati di Weinberg.

(*) Traduzione a cura della Redazione.

Вычисление длин π - π рассеяния с помощью алгебры токов.

Резюме (*). — Предлагается новый подход для вычисления длин π - π рассеяния, основанный на применении алгебры токов. Параметризация, выбранная для амплитуды, учитывает упругие унитарные разрезы. Было найдено семейство решений, которое содержит результаты Вейнберга.

(*) Переведено редакцией.

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